Business Calculus Test 1 Review

Dr. Graham-Squire, Spring 2017

•The test will cover everything in the course up through Section 3.5.

•To study, you should look over your notes, rework HW problems on WebAssign, quizzes, and problems from the notes, as well as work out the practice problems given for each section. The Review Questions at the end of Chapters 1, 2 and 3 are also good practice (though some of them cover other material).

•The following questions from Fall 2014 match with this test: Minitest 1 (all); Test 2: #1-8

•The following questions from Fall 2012 match with this test: Minitest 1 (all); Test 2: all

•The following questions from Summer 2012 match with this test: Test 1: all; Test 2: #4, 6, 7, and 8

•All the questions from Summer 2008, Test 1, match this test.

•Even though we know the shortcut methods to take derivatives, you may be asked to take a derivative using the limit definition on this test. SO YOU MUST KNOW HOW TO DO THE 4-STEP PROCESS FOR THIS TEST.

•Calculators <u>are</u> allowed on this test, but for certain questions you may not be allowed to use a calculator.

•Some practice problems to work on:

1. Factor and simplify the expression $5x^2(3x^2+1)^4(6x)+(3x^2+1)^5(2x)$.

2. The average speed of a vehicle in miles per hour over a certain stretch of road, between 6 AM and 8 AM, is given by
$$20t^2-40t+50 \qquad (0\le t\le 2)$$
where t is in hours, with $t=0$ corresponding to 6 AM. Over what interval(s) of time is the average speed less than or equal to 35 mph?

3. Dominic has decided to attach his baby sister Eva to a rocket. Suppose Dominic is standing 200ft from the rocket (over flat ground) and the rocket with Eva attached lifts off vertically in the air and reaches an altitude of x feet.

   (a) Find an expression (in terms of x) giving the distance between Dominic and Eva as the rocket goes up.

   (b) What is the distance between Dominic and Eva when the rocket reaches an altitude of 1,000 feet? Round to the nearest whole number.

4. Find the equation of the line that passes through the points (1,2) and (-3,-2), and write the equation in the form $y=$ _____. Is the point (4, 8) also on that line? Explain why or why not.

5. Sketch the graph of the function $f(x) = \begin{cases} 4 - x & \text{if } x < 2 \\ 2x - 2 & \text{if } x \geq 2 \end{cases}$

6. Find the rules for the composite functions $f(g(x))$ and $g(f(x))$ if $f(x) = \dfrac{x}{x^2 + 5}$ and $g(x) = \dfrac{1}{x^2}$. Simplify the resulting expressions.

7. The sales of DVD players in year t (in millions of units) is given by the function

$$f(t) = 5.6(1 + t) \qquad (0 \leq t \leq 3)$$

where $t = 0$ corresponds to 2001. Over the same period, the sales of VCRs (in millions of units) is given by

$$g(t) = \begin{cases} -9.6t + 22.5 & \text{if } 0 \leq t \leq 1 \\ -0.5t + 13.4 & \text{if } 1 < t \leq 2 \\ -7.8t + 28 & \text{if } 2 < t \leq 3 \end{cases}$$

(a) Show that more VCRs than DVD players were sold in 2001.

(b) When did sales of DVD players first exceed those of VCRs?

8. Farmer Bob wants to enclose a rectangular area next to his barn with fencing. One side of the rectangular area will be the side of the barn, and the other three sides will be fencing. Suppose that Bob has 80 feet of fencing and his barn is 100 feet long.

(a) If the width of the rectangular area is x, find an expression for the area of the rectangle.

(b) If the width is 10 feet, what will the area of the rectangle be?

9. Find the limits. If the limit does not exist, write DNE and explain why.

(a) $\lim\limits_{x \to 2} \dfrac{x^2 + 4x - 12}{x^2 - 2x}$.

(b) $\lim\limits_{x \to (-1)} \dfrac{x^2}{x + 1}$.

(c) $\lim\limits_{x \to \infty} \dfrac{3x^4 - 3x}{7x^2 - 11x^4 + 4}$.

10. Let $f(x) = \begin{cases} 2x + 3 & \text{if } x < -1 \\ x^2 & \text{if } -1 \leq x \leq 2 \\ 3 & \text{if } x > 2 \end{cases}$

Find the value of the following limits. If the limit does not exist, write DNE and explain why.

(a) $\lim\limits_{x \to (-1)^-} f(x)$.

(b) $\lim\limits_{x \to (-1)^+} f(x)$.

(c) $\lim\limits_{x \to 2^-} f(x)$.

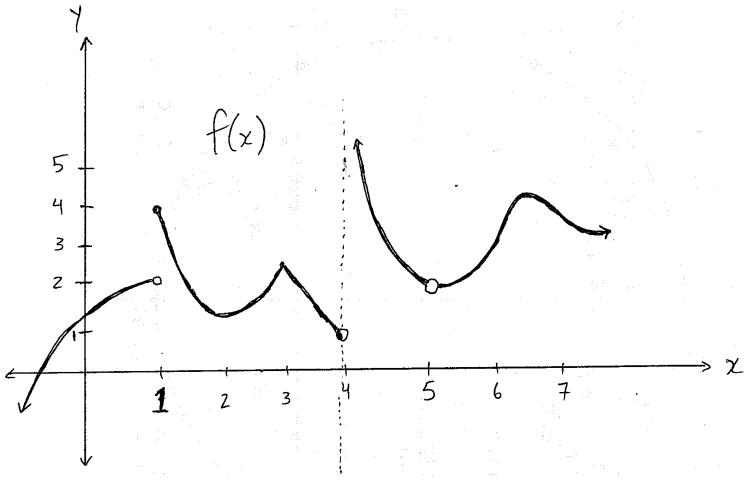
(d) $\lim\limits_{x \to 2^+} f(x)$.

(e) $\lim\limits_{x \to 1^-} f(x)$.

(f) $\lim\limits_{x \to \infty} f(x)$.

11. Use the limit definition of the derivative to calculate $f'(x)$ if $f(x) = \dfrac{1}{2x + 3}$.

12. Find the given limits for the following diagram. If the limit does not exist, write DNE and explain why.



(a) $\lim_{x\to 1^+} f(x) =$

(b) $\lim_{x\to 4^-} f(x) =$

(c) $\lim_{x\to 4^+} f(x) =$

(d) $\lim_{x\to 5} f(x) =$

13. Use derivative rules to find the derivative of each function:

(a) $f(x) = (3x^4 - 7)(x^2 + 9)$. Simplify your answer.

(b) $f(x) = (x^3 - 7x + 9)^7$. You do not need to simplify.

(c) $f(x) = \left(\dfrac{x^3 - 9}{x + 4}\right)^3$. Simplify your answer by combining like terms.

(d) $f(x) = (x + 7)^4(3x^2 - 4)^2$. Simplify your answer by factoring completely.

14. The quantity x of TV sets demanded each week is related to the wholesale price by the equation $p = -0.006x + 180$. The weekly total cost for producing x sets is given by $C(x) = 0.00002x^3 - 0.02x^2 + 120x + 60,000$.

(a) Find the revenue function $R(x)$ and the profit function $P(x)$.

(b) Compute the marginal revenue, marginal cost, and marginal profit functions.

(c) Compute $R'(2000)$, $C'(2000)$, and $P'(2000)$ and interpret your results. What does that information tell the company about how many TV sets they should produce?

15. The number of people receiving disability benefits from 1990 through 2000 is approximated by the function

$$N(t) = 0.00037t^3 - 0.0242t^2 + 0.52t + 5.3 \qquad (0 \le t \le 10)$$

where $N(t)$ is measured in units of a million and t is measured in years with $t = 0$ being 1990. Compute $N(8)$, $N'(8)$, and $N''(8)$ and interpret your results. What does that information tell you about what was happening with disability benefits at that time, and what might it imply for the future?